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DECISION MAKING UNDER FUZZY ENVIRONMENT FOR DETERIORATING ITEMS WITH STOCK DEPENDENT DEMAND UNDER INFLATION EFFECT

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ABSTRACT

This paper discusses a deterministic EOQ (Economic Order Quantity) model for deteriorating items under inflation in a fuzzy environment. The supplier gives some credit period to the retailer if the retailer orders a large quantity. This paper considers the stock dependent demand in which shortages are not allowed. We study the effects of inflation rate, deterioration rate and trade credit period on the optimal cyclic length, optimal order quantity and the total relevant cost by taking costs involved in this paper as triangular fuzzy numbers. A deep study of sensitivity analysis is made to elucidate the fuzzy model better than crisp model. A graded mean representation method is used to de-fuzzify the model.

KEYWORDS: Stock Dependent Demand, Trade Credit Period, Cycle Length, Triangular Fuzzy Numbers

1. INTRODUCTION

In real life situation there is an inventory loss by deterioration. Owing to this fact, how to control and maintain inventories of deteriorating items becomes an important problem for decision maker in modern organization. In practice deterioration of an item is a common phenomenon. Many products in the real world are subject to significant rate of deterioration. Hence the impact of product deterioration should not be neglected in the decision process. During past few decades, many researchers have studied inventory models for deteriorating items such as volatile liquids, blood bank, medicine, electronic items and fashion goods. Some of these products don't have expiration date, they can be stored indefinitely though they suffer natural attrition while being held in inventory. The decrease or loss of utility due to decay is usually a function of on hand Inventory.

Many complicated problems arising in the field of engineering, social science, economics, medical science and management etc involving uncertainties dealt with classical methods are found to be inadequate in recent times. So fuzzy set theory can be used in decision making process for better understanding of uncertainity situation.

2. LITERATURE SURVEY

Large quantity of goods displayed in market lure the customers to buy more. If the stock is insufficient customer may prefer some other brands, as a result it will fetch loss to the supplier. Chung-Yuan Dye[4] developed an inventory model for deteriorating items with stock dependent demand under conditions of permissible delay in payments. Ghare and Schrader [8] were the first proponents to establish a model for an exponentially decaying inventory. Covert and Philip[5] extended Ghare and Schrader's [8] constant deterioration rate to a two-parameter weibull distribution. There has been several research papers on inventory models for deterioration items such as written by Shah and Jaiswal[12] ,Aggarwal[3] , Dave and Pater[7], Sachan[11], Hariga[10] and Goyal and Giri[9].

Most of the classical inventory models did not take into account the effects of inflation. This has happened mostly because of the belief that the inflation would not influence the inventory policy variables to any significant rate. However, most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money in the last several years. Liao et al.,[14] presented a model with deteriorating items under inflation when a delay in payments is permissible. There are several researchers like Brahmbhatt[4]. Chandra and Bahner[6], Datta and pal[9] also considered this area. The traditional EOQ model assumes that the retailer's capitals are unrestricting and must be paid for the items as soon as the items were received. However, this may not be true. In reality, the supplier will offer the retailer a delay period, that is, the trade credit period(Permissible delay in payments) for settling the accounts if the retailer supplier. 1. It will attract new customers who consider it to be a type of price reduction. 2. It will cause a reduction advantage of permissible delay more frequently. Chun-Tao Chang[7] developed an inventory model with deteriorating items for constant demand under inflation and the condition of permissible delay in payments. M. Vallianthal et al. developed the model in which demand rate is a function of stock on hand under inflation effects. The proposed model develops fuzzy EOQ model for deteriorating items with stock dependent demand under inflation effects with costs involved are taken as Fuzzy triangular numbers, as Fuzzy set theory can be considered as a mathematical tool for uncertainities.

3. NOTATIONS

- H – The length of planning horizon
- D(t) – Demand rate (functional)
- T –The replenishment time
- I(t) – The inventory level at time t , $0 \leq t \leq T$
- i – constant rate of inflation / unit time where $0 \leq i \leq 1$
- Pe^{it} –The unit purchasing cost at time t
- $P^{|} e^{it}$ – The selling price/unit at time t where $P^{|}$ uit selling price at time zero
- h- The holding cost rate /unit time excludes interest charges
- Ke^{it}- The ordering cost / order at time t where K is the ordering cost at time zero
- I_e- Interest earned / $ / year.
- I_r- The interest charges / $ in stocks / year by the supplier
- M – The permissible delay in settling accounts (trade credit period)
- Q – The order Quantity
- Q_m – The minimum order quantity at which the delay in payments is permitted.
- T_m – The time interval that Q_m units are deepened to zero due to both demand and deterioration
- ϴ – constant rate of deterioration, $0 < \Theta < 1$
- N – Number of replenishment during H (i.e H = nT)
- TC_i – The total relevant cost over (0 , H) in case where I = 1,2,3,4.

- $\tilde{P}$ e^{it} - fuzzy purchasing cost /unit
- $\tilde{P}^{|}$ e^{it} – Fuzzy selling cost / unit
- $\tilde{h}$ – Fuzzy holding cost / unit time
- $\tilde{K}$ e^{it} -Fuzzy ordering cost / order at time t
- $\tilde{I}_e$ - Fuzzy interest earned in \$/ year
- I_r – The Fuzzy charge in \$ in stocks / year.
- $\tilde{T}C_i$ –Fuzzy total relevant cost.
- $F_i(T)$ – De-fuzzified total relevant cost.
- d_i*- de-fuzzified optimum cycle length
- q_i*- de-fuzzified optimum order quantity

4. ASSUMPTIONS

The Basic assumptions about the model are:

- The demand rate function D(t) is deterministic and is functional given by $D(t) = \alpha+\beta I(t)$, $\alpha > 0$, $0<\beta<1$.
- Constant inflation rate
- Shortages not allowed.
- Instantaneous Replenishment.
- Time Horizon finite.
- If the order quantity is less than Q_m , then the payment for the items received must be made immediately.
- If order quantity is greater than or equal to Q_m, then the delay is payments upto M is permitted. During the trade credit period the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of permissible delay, the customer pays off all the units ordered and starts paying for the interest charges on the in stock items.

5. PROPOSED INVENTORY MODEL IN CRISP SENSE

First, we deal an inventory model without shortage in crisp environment.

Due to combined effects of demand and deterioration in the interval [O,T], the level of inventory gradually decreases.

Variation of Inventory is given by the following differential equation.

$$\frac{dI(t)}{dt} + \theta I(t) = -\alpha - \beta I(t)$$

With the boundary conditions.

I(0) = Q , I(T) = 0

$$I(t)=\left(\frac{\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)(T-t)}-1\right]$$

Order quantity using boundary condition

$$Q=\left(\frac{\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)(T)}-1\right] \tag{1}$$

Simillarly Qm units depleted to zero due to both demand and deterioration.

$$Q_m=\left(\frac{\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)(T_m)}-1\right] \tag{2}$$

$$T_m=\left(\frac{1}{\theta+\beta}\right)\log\left[Q_m\frac{(\theta+\beta)}{\alpha}+1\right] \tag{3}$$

Since the length of time intervals are all the same , we have

$$I(jT+t)=\left(\frac{\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)(T-t)}-1\right] \quad 0\leq j\leq n-1 \quad , 0\leq t \leq T$$

Total relevant cost = Cost of placing orders + cost of purchasing + cost of carrying inventory excluding interest charges + cost of interest charges for unsold items at the initial time or after permissible delay M + the interest earned from sales revenue during permissible period. (4)

Case (1): 0 < T < T_m

Since the replenishment time interval T is less than T_m , the order quantity Q is less than Q_m. So delay in payment is not permitted. Total relevant cost includes only first 4 cost in

$$TC_1(T)=\left\{K+\left(\frac{P\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)T}-1\right]+P\alpha\frac{(h+I_r)}{(\theta+\beta)^2}\left[e^{(\theta+\beta)T}-(\theta+\beta)T-1\right]\right\}\left[\frac{(e^{iH}-1)}{e^{iT}-1}\right]$$

Case (2): $T_m \leq T < M$

Since $T_m \leq$ T < M , there is a permissible delay M which is longer than the replenishment interval T. As a result there is no interest charged, but the interest earned is (0,H).

$$TC_2(T)=\left\{K+\left(\frac{P\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)T}-1\right]+\left[\frac{hp\alpha}{(\theta+\beta)^2}\right]\left[e^{(\theta+\beta)T}-(\theta+\beta)T-1\right]\right.$$

$$P^{|}I_e\left\{\left\{\frac{\alpha T^2}{2}+\frac{\alpha\beta}{(\theta+\beta)^2}\left[e^{(\theta+\beta)T}-(\theta+\beta)T-\frac{T^2(\theta+\beta)^2}{2}\right]\right\}\right\}$$

$$+\left\{(M-T)\left[\alpha T+\frac{\alpha\beta}{(\theta+\beta)^2}\left[e^{(\theta+\beta)T}-1-(\theta+\beta)\right]\right]\right\}\times\left[\frac{(e^{iH}-1)}{e^{iT}-1}\right]$$

Case (3): $T_m \leq M < T$

In this case the replenishment interval T is longer than or equal to both T_m and M. Therefore, the delay in payments is permitted and the total relevant cost includes both the interest charged and the interest earned.

$$TC_3(T)=\left\{K+\left(\frac{P\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)T}-1\right]+\left[\frac{hp\alpha}{(\theta+\beta)^2}\right]\left[e^{(\theta+\beta)T}-(\theta+\beta)T-1\right]\right.$$

$$\mathrm{P}^{|}I_e\left\{\left\{\frac{\alpha M^2}{2}+\frac{\alpha\beta}{(\theta+\beta)^2}\left[e^{(\theta+\beta)T}-M(\theta+\beta)e^{(\beta+\theta)(T-M)}-e^{(\beta+\theta)(T-M)}-\frac{M^2(\theta+\beta)^2}{2}\right]\right\}\right\}$$

$$+\left\{\frac{PI_r\alpha}{(\beta+\theta)^2}\left[\left[e^{(\theta+\beta)(T-M)}-1-(T-M)(\theta+\beta)\right]\right]\right\}\times\left[\frac{(e^{iH}-1)}{e^{iT}-1}\right]$$

Case (4): $M \leq T_m \leq T$

In this case, the replenishment time interval T is also greater than or equal to both T_m and M. Thus, case 4 is similar to case 3.

Optimum value of t in the above cases can be obtained by equating their first order derivatives to zero.

$$T_1^*=\frac{iK\pm\sqrt{(iK)^2+2KP\alpha(h+\theta+\beta+I_r-i)}}{P\alpha(h+\theta+\beta+I_r-i)} \quad \text{....Case (1)}$$

Also $\dfrac{d^2[TC_i(T)]}{dT^2}>0$ i=1,2,3,4.

T_i* is the optimal value of T for case i=1,2,3,4

$$Q(T_i^*)=\left(\frac{\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)(T_i^*)}-1\right]$$

Is the optimal order quantity for cases i=1,2,3,4

6. PROPOSED INVENTORY MODEL IN FUZZY SENSE

We consider the model in Fuzzy environment. Since the costs involved are fuzzy in nature. We represent them by triangular fuzzy numbers.

Case(1) $0 < T < T_m$

$$\tilde{T}C_1(T)=\left\{\tilde{K}\oplus\left(\frac{\tilde{P}\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)T}-1\right]\oplus\tilde{P}\alpha\frac{(\tilde{h}\oplus\tilde{I}_r)}{(\theta+\beta)^2}\left[e^{(\theta+\beta)T}-(\theta+\beta)T-1\right]\right\}\left[\frac{(e^{iH}-1)}{e^{iT}-1}\right]$$

Where $\tilde{K}$ = (K_1, K_2,K_3) , $\tilde{P}$ = (P_1, P_2,P_3)

$\tilde{h}$ = (h1, h2,h3) , $\tilde{I}_r$ = (Ir1, Ir2,Ir3)

$\tilde{P}^{|}$ = ($\tilde{P}^{|}{}_1$, $\tilde{P}^{|}{}_2$, $\tilde{P}^{|}{}_3$)

Defuzzifying $\tilde{T}$ C1(T) by using Graded mean representation method we get.

$$F_1(T) = \frac{1}{6}\Big\{ \mathrm{K}_1 + 4K_2 + K_3 + \alpha P_1(T + h_1 + \theta + \beta + Ir_1)\frac{T^2}{2}$$

$$+4\alpha P_2((T + h_2 + \theta + \beta + Ir_2)\frac{T^2}{2} + \alpha P_3((T + h_3 + \theta + \beta + Ir_3)\frac{T^2}{2}\Big\}\left[\frac{2(e^{iH}-1)}{i(2T + iT^2)}\right]$$

F1(T) is optimum when $\frac{dF_1(T)}{dT} = 0$ and

$$\frac{d^2F_1(T)]}{dT^2} > 0$$

Optimum cycle length for this case is

$$d_1^* = \frac{i(K_1 + 4K_2 + K_3) \pm \sqrt{x\{(P_1(h_1 + \theta + \beta + Ir_1 - i) + 4P_2(h_2 + \theta + \beta + Ir_2 - i) + P_3(h_3 + \theta + \beta + Ir_3 - i)\ \}}}{\alpha\{(P_1(h_1 + \theta + \beta + Ir_1 - i) + 4P_2(h_2 + \theta + \beta + Ir_2 - i) + P_3(h_3 + \theta + \beta + Ir_3 - i)\ \}}$$

Where x= $i^2(K_1 + 4K_2 + K_3)^2 + 2((K_1 + 4K_2 + K_3)\alpha$

$$q_1{*} = \left(\frac{\alpha}{\theta + \beta}\right)\left[e^{(\theta+\beta)(d_1^*)} - 1\right]$$

Case (2): $T_m \leq$ T < M

$$F_2(T) = \frac{1}{6}\Big\{ \mathrm{K}_1 + 4K_2 + K_3 + \alpha P_1(T + (h_1 + \theta + \beta + Ir_1)\frac{T^2}{2})$$

$$4\alpha P_2((T + (h_2 + \theta + \beta + Ir_2)\frac{T^2}{2}) + \alpha P_3((T + (h_3 + \theta + \beta + Ir_3)\frac{T^2}{2}) - (P_1'Ie_1 + 4P_2{}'Ie_2 + P_3{}'Ie_3) \times (\frac{\alpha T^2}{2} + (M - T)(\alpha T + \frac{\alpha\beta T^2}{2})\Big\}\left[\frac{2(e^{iH}-1)}{i(2T + iT^2)}\right]$$

Optimum cycle length for this case is

$$d_2^* = \frac{i(K_1 + 4K_2 + K_3) \pm \sqrt{\begin{aligned}&x\{(P_1(h_1 + \theta + \beta - i) + P_1'Ie_1(1 + Mi) + 4P_2(h_2 + \theta + \beta - i) + 4P_2{}'Ie_2(1 + Mi)\\ &+ P_3(h_3 + \theta + \beta - i) + P_3'Ie_3(1 + Mi)\ \}\end{aligned}}}{\begin{aligned}&\alpha\{(P_1(h_1 + \theta + \beta - i) + P_1'Ie_1(1 + Mi) + 4P_2(h_2 + \theta + \beta - i) + 4P_2{}'Ie_2(1 + Mi)\\ &+ P_3(h_3 + \theta + \beta - i) + P_3'Ie_3(1 + Mi)\ \}\end{aligned}}$$

Where x= $i^2(K_1 + 4K_2 + K_3)^2 + 2((K_1 + 4K_2 + K_3)\alpha$

$$q_2{*} = \left(\frac{\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)(d_2^{*})} - 1\right]$$

Case (3): $T_m \le M < T$

$$F_3(T) = \frac{1}{6}\Bigg\{ \mathrm{K}_1 + 4K_2 + K_3 + \alpha P_1(T + (h_1 + \theta + \beta + Ir_1)\frac{T^2}{2})$$

$$\left.\begin{array}{l} 4\alpha P_2((T + (h_2 + \theta + \beta + Ir_2)\frac{T^2}{2}) + \alpha P_3((T + (h_3 + \theta + \beta + Ir_3)\frac{T^2}{2}) - (P_1' Ie_1 + 4P_2' Ie_2 + P_3' Ie_3) \times \\ \alpha M\left(M - \beta(T-M)^2\right) + \alpha(P_1 Ir_1 + 4P_2 Ir_2 + P_3 Ir_3)(T-M)^2/2 \end{array}\right\}\left[\frac{2(e^{iH}-1)}{i(2T+iT^2)}\right]$$

Optimum cycle length for this case is

$$d_3^{*} = \frac{y \pm \sqrt{\begin{array}{l} y^2 + 4\alpha\{(P_1(h_1 + \theta + \beta - i + Ir_1(1+Mi)) + 4P_2(h_2 + \theta + \beta - i + Ir_2(1+Mi)) \\ + P_3(h_3 + \theta + \beta - i + Ir_3(1+Mi)) \ \} \end{array}}}{\begin{array}{l} 2\alpha\{(P_1(h_1 + \theta + \beta - i + Ir_1(1+Mi)) + 4P_2(h_2 + \theta + \beta - i + Ir_2(1+Mi)) \\ + P_3(h_3 + \theta + \beta - i + Ir_3(1+Mi)) \ \} \end{array}}$$

Where y= $\{\ 2(K_1 + 4K_2 + K_3) + \alpha M^2\left\{\left(P_1 Ir_1 - P_1' Ie_1\right) + 4\left(P_2 Ir_2 - P_2' Ie_2\right) + \left(P_3 Ir_3 - P_3' Ie_3\right)\right\} i \ \}$

$$q_3{*} = \left(\frac{\alpha}{\theta+\beta}\right)\left[e^{(\theta+\beta)(d_3^{*})} - 1\right]$$

Case (4): $M \le T_m \le T$

In this case, the replenishment time interval T is also greater than or equal to both T_m and M. Thus, case 4 is similar to case 3.

7. ALGORITHM FOR FINDING FUZZY TOTAL COST AND OPTIMAL CYCLE LENGTH AND RELEVANT ORDER QUANTITY

Step 1: Calculate total cost for the crisp model as given by equation (4).

Step 2: Use Graded Mean Integration method for defuzzification of $\widetilde{TC}_i$. Then find optimal cycle length d_i*, i=1,2,3,4 and optimal order quantity.

Step 3: Find defuzzified total relevant cost.

8. NUMERICAL EXAMPLE

Crisp Model

Let H= 1 year, α =1000, β=0.1, h=$2/unit/year, I_r =0.09/$/year, I_e=0.06/$/year, i=0.05/unit, P = $20/unit, $P^{|}$=$35/unit, Ɵ =0.03, K =$125/order, Q_m=70 units. Now T_m=0.0697, we take M=15,30 or 45 days. We obtained optimal solutions as follows.

Sensitivity analysis on M

Table 1

Credit Period M	Replenishment Cycle Length T^*	EOQ $Q(T^{*)}$	Total Relevant Cost $TC(T^*)$
15	T_4^*=0.0786	78.3767	23801
30	T_2^*=0.0775	77.8917	23716
45	T_2^*=0.0758	76.1747	23626

It can be seen that higher value of credit period implies lower values of total relevant cost, order quantity and the replenishment

Sensitivity analysis on Inflation rate i

Table 2

Inflation Rate i	Replenishment Cycle Length T^*	EOQ $Q(T^{*)}$	Total Relevant Cost $TC(T^*)$
0.05	T_1^*=0.0680	68.3014	23524
0.10	T_1^*=0.0689	69.2095	24086
0.15	T_2^*=0.0759	76.275	24411

It can be seen that higher value of inflation rate implies large values of total relevant cost, order quantity and the replenishment.

De-fuzzified values

Sensitivity analysis on M

Table 3

Credit Period M	Replenishment Cycle Length d^*	EOQ q^*	Total Relevant Cost $F(q^*)$
15	T_4^*=0.0319	33.57	18501
30	T_2^*=0.0317	33.48	18488
45	T_2^*=0.0315	33.35	18470

Sensitivity analysis on Inflation rate i

Table 4

Inflation Rate i	Replenishment Cycle Length d^*	EOQ q^*	Total Relevant Cost $F(q^*)$
0.05	T_1^*=0.0786	67.304	21500
0.10	T_1^*=0.079	68.108	21789
0.15	T_2^*=0.0801	76.275	22014

9. CONCLUSIONS

In this paper the endeavour of our model is to minimize retailers total inventory cost through various parameters like permissible delay in payments minimum order quantity at inflation rate in both crisp and fuzzy environment by using triangular fuzzy numbers for the relevant cost involved in this model. Defuzzification has been carried out and the corresponding changes have been observed. Finally we conclude that through sensitivity analysis that the results obtained are much better and economic than the case of crisp model when compared with Fuzzy model.

The given concepts is easily applied in actual business scenario.

REFERNECES

1. Inventory systems by Eliezer Naddor
2. Fuzzy sets and logics by Zadeh
3. Agarwal, S.P., 1978. A note on an order level inventory model for a system with constant rate of deterioration, Opsearch 15, 184-187.
4. Chung-Yuan dye, 2002. A Deterioriating Inventory Model with stock-Dependent Demand and partial backlogging under conditions of permissible delay in payments, Opsearch 39, 189-201.
5. Covert, R.B. Philip, g.S., 1973. An EOQ model with weibull distribution deterioration. AIIE Transactions 5, 323-326
6. R.E. Bellman, L.A Zadeh, "Decision- making in a fuzzy environment", Management science, 17(4), B141-B164, 1970.
7. Dave, U. Patel, L.K., 1981 (T ,S) Policy inventory model for deteriorating items with time proportional demand, Journal of the Operational Research Society 32, 137-142.
8. Ghare. P.M., Schrader, G.P., 1963. A model for an exponentially decaying inventory, Journal of Industrial Engineering 14, 238-243.
9. Goyal, S.K., B.C., 2001. Recenttrends in modeling of Deteriorating Inventory, European Journal of Operational Research 134, 1-16
10. Hariga, M.A., 1996, Optimal EOQ models for deteriorating items with time varying demand, Journal of the Operation Research Society 47, 1228-1246.
11. Sacha n.R.S., 1984. On (T.S) policy inventory model for deteriorating items with time proportional demand, Journal of the Operational Research Society 35, 1013-1019.
12. Sha h, Y.K., Jaiswal, M.c., 1977, An order level inventory model for a system with constant rate of deterioration, Opsearch 14, 174-184.